Automatic Reordering for Dataflow Safety of Datalog

Or how I stopped worrying about syntactic order of execution and love greedy scheduling

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The problem with syntactic order of execution

Version 1:
auth(User) :- hash(Pass,Hash), password(User,Pass), valid(User,Hash).

Version 2:
auth(User) :- password(User,Pass), hash(Pass,Hash), valid(User,Hash).
➤ Datalog recap

➤ Modes, adornments & well-modedness

➤ Intra- and inter-clausal analysis

➤ Properties of the analysis

➤ Future work
Datalog recap

- Good for deductive databases, AI, data integration, program analysis
- No function symbols, unlike Prolog, e.g., no lists
- Negation
- Aggregation
- Extralogical predicates, e.g., IO and foreign functions
Example Datalog program

- A set of extensional predicates and intensional predicates.

  pc_predecessor("Brigitte Pientka", "Peter Thiemann").
  pc_predecessor("Germán Vidal", "Brigitte Pientka").
  pc_predecessor("Elvira Albert", "Germán Vidal").
  pc_predecessor("Olivier Danvy", "Elvira Albert").

  ancestor(X,Y) :- pc_predecessor(X,Y).
  ancestor(X,Z) :- pc_predecessor(X,Y), ancestor(Y,Z).

- Queries to retrieve information

  ?- ancestor(X,"Peter Thiemann").
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- Future work
Practical applications require something extra

- Promoting a C function into a logical predicate:

  ```c
  char* function hash(char* content);
  hash(Content, Hash).
  ```
Modes to capture static dataflow

- Use + to require the variable to be bound invocation time
- Use ? to say you do not care if it is bound or not.

- Earlier predicate with mode annotation:
  hash+?(Content, Hash)

- Multiple implementations lead to multiple mode patterns.
Adornments to capture dynamic dataflow

- The binding of variables depends on the query
- Bound variables are marked with $b$ and free ones with $f$
- Traditionally computed left-to-right in clause body

```
?-auth("Rebecca").
auth_b(User) :- hash^{ff}(Pass,Hash), password_{bb}(User,Pass), valid_{bb}(User,Hash).
```

Reordering changes binding pattern:

```
auth_b(User) :- password_{bf}(User,Pass), hash^{bf}(Pass,Hash), valid_{bb}(User,Hash).
```
Well-modedness

Informally, a well-moded program’s subgoals do not give invocation errors due to insufficient argument binding.

Formally, an agreement between the mode patterns and the adornment of subgoals.

Consider the two adornments of hash:

\[ \text{hash}^{\gamma_f \text{ff}}(\text{Pass, Hash}) \] ✗

\[ \text{hash}^{\gamma_f \text{bf}}(\text{Pass, Hash}) \] ✓
Global reordering is needed

- Recall the different orderings of authentication clauses

  \[
  \text{auth}_b(\text{User}) :- \text{hash}^+\text{ff}(\text{Pass},\text{Hash}), \text{password}_{bb}(\text{User},\text{Pass}), \text{valid}_{bb}(\text{User},\text{Hash}).
  \]

  \[
  \text{auth}_b(\text{User}) :- \text{password}_{bf}(\text{User},\text{Pass}), \text{hash}^+\text{bf}(\text{Pass},\text{Hash}), \text{valid}_{bb}(\text{User},\text{Hash}).
  \]

- What if it was written this way?

  \[
  \text{auth}_b(U) :- \text{check}_{bf}(U,P), \text{password}_{bb}(U,P).
  \]

  \[
  \text{check}_{bf}(U,P) :- \text{hash}^+\text{ff}(P,H), \text{valid}_{bb}(U,H).
  \]

- Reordering the caller help well-moding the subgoals of callee!

  \[
  \text{auth}_b(U) :- \text{password}_{bf}(U,P), \text{check}_{bb}(U,P).
  \]

  \[
  \text{check}_{bb}(U,P) :- \text{hash}^+\text{bf}(P,H), \text{valid}_{bb}(U,H).
  \]
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Mode analysis in two parts

- *Intra-clausal analysis* determines an ordering constraint for each clause based on its subgoals and known constraints alone.

- *Inter-clausal analysis* updates constraints until they stabilise (a fixpoint is reached).
Intra-clausal analysis

- Be greedy and schedule easy subgoals ASAP
- Exploit shared variables between subgoals
- Produce orderings using a graph construction that encodes orderings as paths
Intra-clausal example

\[ r(Y,Z) :\text{=} f^+(X), \ g^{++,++}(X,Y,Z), \ h^+(Z), \ i(X), \ j(X,W). \]

\[ A = (\{X, Y, Z, W\}, \{ (f, \{X\}), (g, \{X, Y\}), (g, \{X, Z\}), (h, \{Z\}), (i, {}), (j, {}) \}, \{\}) \]
Intra-clususal example

\[ r(Y,Z) : - f^+(X), g^{++?,+?+}(X,Y,Z), h^+(Z), i(X), j(X,W). \]

\[
A = (\{X, Y, Z, W\}, \{(f, \{X\}), (g, \{X, Y\}), (g, \{X, Z\}), (h, \{Z\}), (i, {}), (j, {})\}, {}) \\
B = (\{Y, Z\}, \{(f, {}), (g, \{Y\}), (g, \{Z\}), (h, \{Z\})\}, \{\})
\]

Cost to pay
Alternatives & their costs
Cost paid
Intra-clusal example

\[ r(Y,Z) :- f^+(X), g^{++,+?}(X,Y,Z), h^+(Z), i(X), j(X,W). \]

\[ A = \{ X, Y, Z, W \}, \{ (f, \{ X \}, (g, \{ X, Y \}, (g, \{ X, Z \}), (h, \{ Z \}), (i, \{ \}), (j, \{ \}) \} \}, \{ \} \]  
\[ B = \{ Y, Z \}, \{ (f, \{ \}), (g, \{ Y \}), (g, \{ Z \}), (h, \{ Z \}) \}, \{ \} \]  
\[ C = \{ Y, Z \}, \{ (g, \{ Y \}), (g, \{ Z \}), (h, \{ Z \}) \}, \{ \} \]
Intra-clusaul example

\[ r(Y,Z) : - f^+(X), g^{+?,+?+}(X,Y,Z), h^+(Z), i(X), j(X,W). \]

A = (\{X, Y, Z, W\}, \{(f, \{X\}), (g, \{X, Y\}), (g, \{X, Z\}),
(h, \{Z\}), (i, {}), (j, {})\}, \{\})

B = (\{Y, Z\}, \{(f, {}), (g, \{Y\}), (g, \{Z\}), (h, \{Z\})\}, {})

C = (\{Y, Z\}, \{(g, \{Y\}), (g, \{Z\}), (h, \{Z\})\}, {})

D = (\{}, \{(h, {}), \{Y\}\})

Cost to pay
Alternatives & their costs
Cost paid
Intra-clausal example

\[ r(Y, Z) : - f^+(X), g^{??,??}(X, Y, Z), h^+(Z), i(X), j(X, W). \]

A = (\{X, Y, Z, W\}, \{ (f, \{X\}), (g, \{X, Y\}), (g, \{X, Z\}), (h, \{Z\}), (i, \{}), (j, \{}), (\{} \})

B = (\{Y, Z\}, \{ (f, \{}), (g, \{Y\}), (g, \{Z\}), (h, \{Z\}), (\{} \})

C = (\{Y, Z\}, \{ (g, \{Y\}), (g, \{Z\}), (h, \{Z\}), (\{} \})

D = (\{\}, \{ (h, \{}), (\{} \})

E = (\{\}, (\{}), (\{} \})
Intra-clausal example

\[ r(Y, Z) \leftarrow f^+(X), g^{++?,+++}(X, Y, Z), h^+(Z), i(X), j(X, W). \]
Intra-clausal example

\[ r(Y,Z) :: - f^+(X), g^{++,++}(X,Y,Z), h^+(Z), i(X), j(X,W). \]

\[
A = (\{X, Y, Z, W\}, \{ (f, \{X\}), (g, \{X, Y\}), (g, \{X, Z\}), (h, \{Z\}), (i, \{}), (j, \{})) \}
\]

\[
B = (\{Y, Z\}, \{(f, \{}), (g, \{Y\}), (g, \{Z\}), (h, \{Z\})\}, \{\})\]

\[
C = (\{Y, Z\}, \{(g, \{Y\}), (g, \{Z\}), (h, \{Z\})\}, \{\})\]

\[
D = (\{\}, \{(h, \{\})\}, \{Y\})\]

\[
E = (\{\}, \{\}, \{Y\})\]

\[
F = (\{\}, \{(h, \{\})\}, \{Z\})\]

\[
G = (\{\}, \{\}, \{Z\})\]
Intra-clausal example

\[ r(Y,Z) : - f^+(X), g^{++?,+?+}(X,Y,Z), h^+(Z), i(X), j(X,W). \]
Intra-clusual example

\[ r(Y,Z) : - f^+(X), g^{+++}(X,Y,Z), h^+(Z), i(X), j(X,W). \]
Intra-clausal example

Head predicate: \( r(Y, Z) \)

\[ E = (\emptyset, \emptyset, \{Y\}) \]
\[ G = (\emptyset, \emptyset, \{Z\}) \]

\[ r^{+?,?+}(Y, Z) \]
Intra-clausal example: Path extraction

Orderings of subgoals leading to ?+

\[ r(Y,Z) : - \ i(X), \ j(X,W), \ f^+(X), \ g^{++?,++?}(X,Y,Z), \ h^+(Z). \]
\[ r(Y,Z) : - \ j(X,W), \ i(X), \ f^+(X), \ g^{++?,++?}(X,Y,Z), \ h^+(Z). \]
\[ r(Y,Z) : - \ j(X,W), \ i(X), \ f^+(X), \ h^+(Z), \ g^{++?,++?}(X,Y,Z). \]
\[ r(Y,Z) : - \ i(X), \ j(X,W), \ f^+(X), \ h^+(Z), \ g^{++?,++?}(X,Y,Z). \]
Inter-clausal analysis

- Constraint of a predicate respects the constraint of each of its clauses

\[
\text{r}(X, Y, Z) :\ p^{+?,+?}(X, Y), \ a(Z). \\
\text{Intra-clausal analysis} \\
\text{r}^{+??,?+?}(X, Y, Z) \oplus \text{r}^{?+}(X, Y, Z) \\
\text{r}^{?+?,??+}(X, Y, Z)
\]

- Update the constraints for each predicate
- Rinse and repeat until a fixpoint is reached
▶ Datalog recap
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▶ Properties of the analysis
▶ Future work
Sound and complete

- **Soundness** says if the algorithm finds an ordering for all clauses, there will not be invocation errors.

- **Completeness** says if there is an ordering of subgoals that eliminates invocation errors, the analysis will find it.
Incremental analysis

- Datalog is interactive, do not want to recompute.

- Addition of rules *never* invalidates previous analysis.

- When new rules do not extend existing predicates, it suffices to analyse just the new rules. Good for libraries.

- A query requires a single intra-clausal analysis round.
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Future work

- User given mode annotations for intentional predicates
- Analysis graphs contain other useful dataflow information
- $\oplus$ and $\otimes$ form Martelli’s semiring suggesting analysis might be reduce to matrix operations
- Inlining and similar optimisations provide further well-moding opportunities
Recap

- Imperative programming tries to sneak in to declarative programming, we can do better!

- Well-modedness for Datalog is fully captured by adornments and simple modes

- It is possible to do better than brute-force search whilst remaining sound and complete
Thanks. Questions?